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13. ABSTRACT (Maximum 200 words) We studied AWGN coding theorems for ensembles of coding systems which are built from fixed convolutional codes interconnected with random interleavers. We call these systems turbo-like codes and they include as special cases both the classical turbo codes and the serial concatenation of interleaved convolutional codes. We offered a general conjecture about the behavior of the ensemble (maximum-likelihood decoder) word error probability as the word length approaches infinity. We proved this conjecture for a simple class of rate $1/q$ serially concatenated codes where the outer code is a $q$ -fold repetition code and the inner code is rate 1 convolutional code with transfer function $1/(1+D)$ . We call these codes "RA" (repeat and accumulate) codes. This was the first rigorous proof of a coding theorem for turbo-like codes.			
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**FINAL TECHNICAL REPORT** on AFOSR grant no. F49620-97-0313:

*"Turbo Decoding of High performance Error-Correcting Codes via Belief Propagation"*

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**OBJECTIVES:** No change from those stated in the original proposal.

**STATUS OF EFFORT:** The work went extremely well at both institutions. We had frequent joint meetings (approx. 1 per month) in which each group informed the other about its progress. Technical details follow.

**ACCOMPLISHMENTS/NEW FINDINGS:**

\*\*\*At Caltech:

We studied AWGN coding theorems for ensembles of coding systems which are built from fixed convolutional codes interconnected with random interleavers. We call these systems "turbo-like" codes and they include as special cases both the classical turbo codes and the serial concatenation of interleaved convolutional codes. We offered a general conjecture about the behavior of the ensemble (maximum-likelihood decoder) word error probability as the word length approaches infinity. We proved this conjecture for a simple class of rate  $1/q$  serially concatenated codes where the outer code is a  $q$ -fold repetition code and the inner code is a rate 1 convolutional code with transfer function  $1/(1+D)$ . We call these codes "RA" (repeat and accumulate) codes. This was the first rigorous proof of a coding theorem for turbo-like codes.

These results show that the performance of RA codes with maximum-likelihood decoding is very good. However, the complexity of ML decoding of RA codes, like that of all turbo-like codes, is prohibitively large. But an important feature of turbo-like codes is the availability of a simple iterative, message passing decoding algorithm that approximates ML decoding. We wrote a computer program to implement this "turbo-like" decoding for RA codes with  $q=3$  (rate  $1/3$ ) and  $q=4$  (rate  $1/4$ ), and the results were remarkably good. For example, with an information block length of 16384, the  $q=4$  RA code achieves a decoded word error probability of about  $10^{-5}$  at  $E_b/N_0 = 0.5$  dB with 20 decoding iterations.

This work demonstrated that there is a much less complex way to achieve near Shannon limit performance than was previously suspected.